EGYPTIAN RUSSIAN UNIVERSITY
FACULTY OF ENGINEERING

SPRING SEMESTER
MATH. (4).

VECTOR ANALYSIS

اول سبع
انطلاقات

SHEET (1)

1- **Find the directional derivative of the scalar field:**

$$\varphi = xy^2 + yz^3$$

at the point (2, -1, 1) parallel to the vector: $\bar{i} + 2\bar{j} + 2\bar{k}$.

2- **Find the unit vector normal to the surface:**

$$xy^3z^2 = 4$$

at the point (-1, -1, 2).

3- **Calculate the projection of the vector** $\overline{\nabla} u$ **on the vector:**

$$\frac{dx}{ds}\bar{i} + \frac{dy}{ds}\bar{j} + \frac{dz}{ds}\bar{k}.$$

4- **Get** $\overline{\nabla}\varphi$ **in the following cases:**

- $\varphi = xyz$
- $\varphi = x^2 + y^2 + z^2$
- $\varphi = (x^2 + y^2 + z^2)^{\frac{1}{2}}$
- $\varphi = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$

5- $(\overline{\nabla}\varphi)^2$ **is defined as:** $(\overline{\nabla}\varphi)^2 = \overline{\nabla}\varphi.\overline{\nabla}\varphi$ **, prove that:**

$$(\overline{\nabla}\varphi)^2 = (\frac{\partial\varphi}{\partial x})^2 + (\frac{\partial\varphi}{\partial y})^2 + (\frac{\partial\varphi}{\partial z})^2$$

6- **Find the directional derivative at the point (1, 2, 3) for the scalar field:**

$$\varphi = x^2 + y^2 + z^2$$

parallel to the straight line: $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$.

7- **What is the value of the maximal rate of change of the function :** $u = xyz^2$ **at the point (1, 0, 3).**

8- **Given the functions :** $u = x + y + z,\quad v = x + y \quad and \quad w = -2xz - 2yz - z^2$
prove that the triple scalar product: $[\overline{\nabla}u, \overline{\nabla}v, \overline{\nabla}w] = 0$.

9- **Find the unit vector normal to the surface:** $x^2y^2z^3 = 8$ **at the point (1, 1, 2).**

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SHEET (2)

1- A vector field $\overline{F}$ is defined as :

$$\overline{F}(x,y) = y.\cos(2x + y^2)\overline{i} - \cos(2x + y^2)\overline{j} + x^2 y\overline{k}$$

prove that this field is solenoidal.

2- Find the curl of the vector field :

$$\overline{A}(x,y,z) = xy^2\overline{i} + (x+y)\overline{j} + x^3 yz\overline{k} .$$

3- Prove that the vector function :

$$\overline{F}(x,y,z) = (xz^2 - xy^2)\overline{i} + (x^2 y - yz^2)\overline{j} + (y^2 z - x^2 z)\overline{k}$$

is solenoidal.

4- Prove that the vector field :

$$\overline{F}(x,y,z) = 2xy^2 z\overline{i} + 2x^2 yz\overline{j} + x^2 y^2\overline{k}$$

is irrotational.

Prove also that there is a scalar potential φ conjugate to the field $\overline{F}$ such that :

$$\overline{F} = -\overline{\nabla}\varphi$$

hence find the mathematical expression for φ .

5- Given a vector field $\overline{F}$ defined as :

$$\overline{F}(x,y,z) = x^3 yz\overline{i} + xy^3 z\overline{j} + xyz^3\overline{k},$$

find its curl.

Prove that this curl represents a solenoidal field.

6- If $\varphi = xyz^2$, find its gradient, hence prove that its curl gradient vanishes

7- If $\varphi = x^2 + y^2 - 2z^2$, prove that div grad$\varphi = 0$.